EVALUATING THE ALTERNATIVE SOLUTIONS OF WALL INSULATION BY MULTICRITERIA METHODS

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Abstract. The economic effect of buildings’ renovation largely depends on implementation of energy saving methods and devices. A very high economic effect is achieved by wall insulation. In this respect, it is even higher than the replacement of windows. The alternative solutions of wall insulation of buildings differ in the materials used, labour expenditure and other aspects. The cost of renovation depends on the solutions made. The criteria describing the available wall insulation alternatives may have different values. Moreover, they may change in different directions, i.e. a higher value of some criteria denotes a better state, while for others they mean a worse situation. In this environment, a compromise variant is required, which can be found by multicriteria evaluation methods (MCEM). To reduce the effect of various methods on calculation results, it can be recommended to assess the object (or phenomenon) considered by several different methods, with a subsequent determination of the average estimate value. In this way, the disadvantages of some particular multicriteria evaluation methods could be compensated by the advantages of others. The integration of methods will be correct if there is a correlation between the values obtained by different methods.

Keywords: renovation of buildings; wall insulation; multicriteria evaluation; consistency of methods.

1. Introduction

In the report on the climate change made in 2007 (International Panel … 2007), it is stated that, according to the data obtained from continents and oceans, most of the ecosystems are affected by the local climate change, and by the rise of temperature, in particular. It is believed that nearly all regions of the world will be negatively affected by the climate changes, which, in turn, will cause problems in most economy sectors. It is of great importance that more than 187 countries charted the course for a new negotiating process on the ways of reducing the consequences of the climate change at the UN Climate change conference (Bali, Dec, 3–14, 2007) (United Nations … 2007).

Gases causing the greenhouse effect (GHG) are produced by transport, industry and agriculture. This is the main cause of recently observed global warming. The emission of GHG in the world has been increasing since the pre-industrial age. In 1970–2004, it increased by 70%. The buildings in European countries consume more than 40% of energy consumed by all EU member-states, with residential buildings using about 63% (Balaras et al. 2007). Methods of saving energy in buildings considerably reduce their energy consumption, thereby reducing GHG emission. Recent research has shown that awareness of the problem exists in most countries of the world. It has also demonstrated a great economic potential which could be used to reduce the emission of GHG in the world in next few decades.

The envelopes of large-panel residential buildings constructed in the years of Soviet power had poor thermal insulation. Therefore, now we face the problem of renovating the deteriorated buildings (Zavadskas et al. 2004). The investigation has shown that thermal transmittance is 1.6–5.85 times of the specified value. This leads to great heat losses and the lack of thermal comfort in premises. To improve the conditions, additional insulation should be installed into external walls of the buildings. The most suitable and effective way to achieve this is the insulation of walls from the outside (Sadauskienė et al. 2007). This may be done by pasting the walls over with insulating materials or fixing them in some other way and then finishing the walls with stucco. Finally, the walls are covered with boards or other elements.

The insulation of walls for warm-keeping consists of a number of consecutive operations, i.e. fixing the heat insulation board to the wall, fixing the reinforcing mesh to the heat insulation board, finishing etc. Each operation requires some particular materials (e.g. insulating boards, adhesive, reinforcing mesh, pins, mortar etc.) and labour input. Various materials, differing in weight, thermal characteristics, durability etc can be used. The choice of building materials determines the cost of thermal insulation of the walls. Under market conditions, when most residential houses are private, the heavy burden of paying for renovation is placed on the owners organizations. Therefore, they are interested in a lower cost of wall insulation. In this context, the choice of a rational alternative of this operation becomes a significant research and prac-
tical problem. The criteria describing the available thermal insulation alternatives for walls may be assessed differently – for some people they can be better, while for others – worse. Moreover, they may change in different directions, i.e. in some cases, the increasing criterion value can indicate a better situation, while in others it means a worse state.

In this environment, a compromise variant can be found by applying multicriteria evaluation methods (Hwang, Yoon 1981; Brauers et al. 2007; Ginevicius, Podvezko 2006a, 2007a; Zavadskas, Kaklauskas 2007; Kaklauskas et al. 2007; Vileikiene, Zavadskas 2007; Zavadskas et al. 2007; Kalibatas et al. 2007; Ginevicius 2006). Whatever method used, the values and weights of the criteria should be known. Various parameters of the materials used can be found in manuals, specifications, etc. The criteria weights should be determined by experts. There are many ways of weight determination (Hwang, Yoon 1981; Zavadskas, Kaklauskas 2007; Zavadskas, Vilutienė 2006; Saaty 1980; Ginevicius et al. 2004, 2007; Ginevicius 2006; Lin et al. 2008). Some of them are not sufficiently accurate because they are too simple, others are too complicated for a practical application. In any case, the accuracy of expert evaluation largely depends on the number of criteria. When this number is growing, a limit can be reached when an expert can no longer compare the alternatives and do mental arithmetic to determine their weights.

The calculations made in the present work by various multicriteria evaluation methods allowed us to identify the most effective building wall insulation alternative out of five considered options.

2. The role of wall insulation in improving operating characteristics of buildings

Wall insulation is aimed at
1. reducing energy consumption;
2. increasing market value of buildings (Zavadskas et al. 2008);
3. improving performance of building structures and increasing service life of a building (which can be increased up to 40 years (Biekša et al. 2006; Sausnaukaite et al. 2007));
4. raising the comfort level in a building;
5. improving architectural solutions of buildings’ facades matching up with the environment.

The renovation of buildings usually includes the operations of:
- increasing roof insulation and providing a new waterproof covering,
- replacement of windows,
- replacement of entrance doors,
- glazing of balconies,
- wall insulation from the outside of the building,
- reconstruction of a heating unit or system.

Before starting the renovation, the efficiency of energy-saving improvements should be calculated. The improvement (measure) will make sense if the value of the energy saved during the building’s service life will exceed the investments into its implementation (Gorgolewski 1995):

\[
\text{SIR} = \frac{\text{current value of energy saving, } Lt}{\text{cost of investments, } Lt} \geq 1, \quad (1)
\]

where \text{SIR} is the efficiency of energy saving improvement.

By using formula (1), the most effective measures of building renovation can be determined. Such calculations were performed for the improvements made in renovating the main building of Vilnius Gediminas Technical University (VGTU CR) (Fig. 1). The improvements were made in the framework of the international project Framework 6 „Bringing Retrofit Innovation to the Application of Public Buildings“ (BRITA in PuBs), funded by the EU (Bringing … 2004).

As shown in Fig. 1, reconstruction of the heating unit and insulation of walls and the roof of a building produced the highest economic effect. Wall insulation is much more energy-effective than the replacement of windows because, in this case, the investment repays in a shorter time. This can be seen from \text{SIR} value which is equal to 1,16. It is 1,76 times the value of \text{SIR} for windows – 0,66.

Before starting the renovation of envelopes of VGTU main building, their thermal insulation characteristics were determined by an infrared camera “Therma CAM B2”. It was found that windows as well as joints between windows and walls and the external wall boards had the highest thermal transmittance. This reaffirmed the idea that the insulation of walls was required.

The effectiveness of insulation of the external walls depends on many factors (Pikutis, Šeduikytė 2006; Nikićin, Lapko 2006): the cost of thermal renovation, adhesive joint strength (concrete/thermal insulating board), thermal transmittance of thermal insulating board (perpendicular to its surface), compressive strength of the mix used in reinforcing, strength of adhesion between the concrete mix used in reinforcing and thermal insulating board, tensile strength of reinforcing fabric, compressive strength of textured finish, water absorption of textured finish, strength of adhesion between textured finish and concrete; the value of the force required to extract the pin.
fixing a thermal insulating board to solid materials, warranty period, service life, time of work execution.

3. Complex quantitative evaluation of the alternative solutions of wall insulation

The main problem in building renovation is the choice of a subcontractor. The requirements to this task are stated in technical specifications of the provided documents of purchasing which should guarantee competition and encourage the candidates to offer the alternative engineering solutions. The customer (client) rejects the offers not complying with the requirements provided in specifications. He evaluates the offers from two perspectives. In the first case, the lowest cost offered is a key criterion, while, in the second, an economically effective scenario is chosen based on a number of criteria, such as quality, cost, technical advantages, aesthetic, functional and environmental characteristics as well as maintenance costs, efficiency, warranty and technical support, execution period etc.

According to Lithuanian laws, the specific weight of the cost as a criterion reflecting economic efficiency of the suggested alternative should be not smaller than (The amendment … 2002):

- 60% – when cost and three or more other criteria are considered;
- 70% – when cost and two other criteria are considered;
- 80% – when cost and one more criterion are considered.

In order to offer scientifically grounded methodology for selecting the most effective wall insulation alternative, a hypothesis was adopted that the wall insulation for warm-keeping is a complex phenomenon which cannot be evaluated on the basis of a single criterion, e.g. the cost of operation. This phenomenon is multifaceted, with each of the facets being described by a particular criterion. In this way, to obtain a true picture, all of them should be integrated into a single criterion. This problem is complicated because the criteria can be expressed in different dimensions. Moreover, they may change in different directions, i.e. the higher value of some criteria denote a better state, while for others they mean a worse situation.

Recently, multicriteria evaluation methods have been successfully used to quantitatively evaluate such complex and controversial phenomena. To apply them, the following procedures should be performed in three steps: a set of criteria describing the object considered should be developed, the criteria weights and significances should be determined and an appropriate multicriteria evaluation method should be chosen. These methods were applied to select the most economically effective alternative of insulating the external walls of the VGTU main building. To develop a set of criteria describing the process of wall insulation, which could be used in choosing the best alternative, a survey of experts from the Certification Centre of Construction Products, as well as specialists from construction and reconstruction enterprises and researchers, was conducted. At the first stage, the experts evaluated 20 criteria describing quality and cost of wall insulation. At the second stage (Ginevičius, Podvezko 2006b, 2007b), the main 9 criteria were selected (Table 1).

<table>
<thead>
<tr>
<th>No</th>
<th>Description of criteria</th>
<th>Unit of measurement</th>
<th>Direction of the criterion variation</th>
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<tbody>
<tr>
<td>1</td>
<td>Cost of wall insulation</td>
<td>Lt</td>
<td>–</td>
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<tr>
<td>2</td>
<td>Adhesive (glued) joint strength $\sigma_{\text{ad}}$ (concrete/thermal insulation board)</td>
<td>N/mm²</td>
<td>+</td>
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<tr>
<td>3</td>
<td>Thermal transmittance of thermal insulation board $\lambda_d$</td>
<td>W/m²K</td>
<td>–</td>
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<tr>
<td>4</td>
<td>Fabric reinforcement weight $G$</td>
<td>gr/m²</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>Water absorption coefficient of textured finish $w_f$</td>
<td>kg/m²h⁰.⁵</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>Extraction force of a pin fixing thermal insulating board to solid materials $F$</td>
<td>kN</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>Warranty period $t_w$</td>
<td>years</td>
<td>+</td>
</tr>
<tr>
<td>8</td>
<td>Service life (longevity) $t_l$</td>
<td>years</td>
<td>+</td>
</tr>
<tr>
<td>9</td>
<td>Time of completion $t_c$</td>
<td>days</td>
<td>–</td>
</tr>
</tbody>
</table>

At the next stage, the values of the criteria used in multicriteria evaluation were determined. They were obtained from the offers provided by the candidates representing the construction enterprises. In general, 7 enterprises provided the tenders. Two of them were rejected as not sufficiently qualified. The bids of the remaining 5 enterprises, with the values of the criteria described, are given in Table 2.

The first contractor offered the solution of wall heating which was the best as far as such criteria as strength of adhesion (concrete/heating insulation board) – $\sigma_{\text{ad}} = 0.5$ N/mm² and work execution time – $t_e = 50$ workdays were concerned. The second contractor offered the lowest cost of 354 050Lt for wall insulation. The best criterion of the third contractor’s offer was the extraction force of the pin attaching heating insulating boards to solid materials – $F = 0.5$ kN. However, the value of the reinforcing fabric weight $G = 170$ gr/m² in this offer was the highest and other criteria were also much worse than those of other bids. The 4th and the 5th contractors failed to offer any criterion better than those of other bidders. Moreover, the cost offered by the 5th contractor was the highest, while other criteria were not good either. The thermal transmittance of thermal insulation board was the highest $\lambda_d = 0.041$ W/m²K, and it was the worst characteristic compared to others. The criteria describing the offer of the first contractor were in the group of the best indicators, with the weight of reinforcing fabric $G = 165$ gr/m² and
The worst criteria of the second contractor’s offer were thermal transmittance \( \lambda_d = 0.038 \text{ W/m}^2\text{K} \), the weight of reinforcing fabric \( G = 165 \text{ gr/m}^2 \), water absorption of textured finish \( w_p = 0.30 \text{ kg/m}^2\text{h}^{0.5} \) and warranty period \( t_w = 7 \) years.

The following criteria characterizing the offer of the 4th contractor were included in the best group: thermal transmittance of thermal insulating board \( \lambda_d = 0.038 \text{ W/m}^2\text{K} \), the weight of reinforcing fabric \( G = 165 \text{ gr/m}^2 \) and water absorption of textured finish \( w_p = 0.30 \text{ kg/m}^2\text{h}^{0.5} \). The best criteria of the 5th contractor’s offer were strength of adhesion (concrete/thermal insulating board) \( \sigma_{int} = 0.1 \text{ N/mm}^2 \), the extraction force of the pin fixing thermal insulating board to solid materials \( F = 0.25 \text{ kN} \) and service life \( t_l = 40 \) years. The worst criteria describing the 4th contractor’s offer were strength of adhesion (concrete/thermal insulating board) \( \sigma_{int} = 0.1 \text{ N/mm}^2 \), the extraction force of the pin fixing thermal insulating board to solid materials \( F = 0.25 \text{ kN} \) and service life \( t_l = 30 \) years. The worst criteria of the 3rd contractor’s offer were water absorption of textured finish \( w_p = 0.35 \text{ kg/m}^2\text{h}^{0.5} \), warranty period \( t_w = 5 \) years and work execution time \( t_c = 70 \) days. The worst criteria describing the fourth contractor’s offer were strength of adhesion (concrete/thermal insulating board) \( \sigma_{int} = 0.1 \text{ N/mm}^2 \), the extraction force of the pin fixing thermal insulation board to solid materials \( F = 0.25 \text{ kN} \), warranty period \( t_w = 5 \) years and work execution time \( t_c = 70 \) days.

The ranks of 5 contractors (enterprises) considered are given in Table 3 according to the values of the criteria.

### Table 3. The ranks obtained by 5 contractors (enterprises) considered

<table>
<thead>
<tr>
<th>No</th>
<th>Description of criteria</th>
<th>Units of measurement</th>
<th>Direction of change</th>
<th>Criteria significance</th>
<th>Wall insulation alternatives</th>
<th>Ultimate rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cost of wall insulation</td>
<td>Lt</td>
<td>–</td>
<td>0.6</td>
<td>Ltd1: 358900, Ltd2: 354050, Ltd3: 383150, Ltd4: 392850, Ltd5: 407400</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Adhesive (glued) joint strength ( \sigma_{int} ) (concrete/thermal insulating board)</td>
<td>N/mm²</td>
<td>+</td>
<td>0.0148</td>
<td>Ltd1: 0.5, Ltd2: 0.1, Ltd3: 0.1, Ltd4: 0.2, Ltd5: 0.12</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Thermal transmittance of thermal insulating board ( \lambda_d )</td>
<td>W/m²K</td>
<td>–</td>
<td>0.084</td>
<td>Ltd1: 0.039, Ltd2: 0.038, Ltd3: 0.039, Ltd4: 0.038, Ltd5: 0.041</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Fabric reinforcement weight ( G )</td>
<td>gr/m²</td>
<td>–</td>
<td>0.008</td>
<td>Ltd1: 165, Ltd2: 165, Ltd3: 170, Ltd4: 165, Ltd5: 165</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Water absorption coefficient of textured finish ( w_p )</td>
<td>kg/m²h^{0.5}</td>
<td>–</td>
<td>0.012</td>
<td>Ltd1: 0.35, Ltd2: 0.30, Ltd3: 0.35, Ltd4: 0.30, Ltd5: 0.35</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>Extraction force of a pin fixing thermal insulating board to solid materials ( F )</td>
<td>kN</td>
<td>+</td>
<td>0.03</td>
<td>Ltd1: 0.25, Ltd2: 0.25, Ltd3: 0.5, Ltd4: 0.25, Ltd5: 0.25</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Warranty period ( t_w )</td>
<td>years</td>
<td>–</td>
<td>0.031</td>
<td>Ltd1: 5, Ltd2: 7, Ltd3: 5, Ltd4: 5, Ltd5: 7</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>Service life (longevity) ( t_l )</td>
<td>years</td>
<td>+</td>
<td>0.039</td>
<td>Ltd1: 40, Ltd2: 30, Ltd3: 35, Ltd4: 30, Ltd5: 40</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>Time of completion ( t_c )</td>
<td>days</td>
<td>–</td>
<td>0.01</td>
<td>Ltd1: 50, Ltd2: 60, Ltd3: 70, Ltd4: 70, Ltd5: 60</td>
<td>4</td>
</tr>
</tbody>
</table>
In such a controversial situation, it is difficult to select the best alternative without using mathematical methods. As mentioned above, to solve such problems multiple evaluation methods should be applied.

The weight values can be used in further multicriteria evaluation, provided that experts’ judgments are consistent (in concordance). The concordance level can be determined by Kendall’s concordance coefficient \(W\) (Kendall 1970; Zavadskas, Vilutienė 2006; Zavadskas, Kaklauskas 2007; Podvezko 2005, 2007; Kaklauskas et al. 2006; Ginevičius et al. 2008). To calculate this coefficient, preliminary ranking of the criteria with respect to each expert should be performed, implying that the most important criterion is given the highest value equal to unity (one), the next most important criterion is given the value of 2, etc., while the least important criterion is given the value \(m\), with \(m\) denoting the number of the criteria considered. Similar estimates are given the same rank, i.e. the arithmetical mean of the respective ranks.

The ranking results of 16 experts’ estimates \(e_{ik}\) (i = 1, 2, ..., \(m\); j = 1, 2, ..., \(r\); \(m\) is the number of the criteria, \(r\) – the number of experts) are in Table 4.

The data on the first criterion are not provided in the table because, as mentioned above, according to Lithuanian laws, the first criterion (cost) is prescribed at least 60% of the significance of all criteria, i.e. the weight of the first criterion is \(\omega_1 = 0.6\).

The concordance coefficient \(W\) is calculated by the formula (Kendall 1970):

\[
W = \frac{12S}{r^2m(m^2 - 1)},
\]  

(2)

where \(r\) is the number of experts, \(m\) – the number of the criteria considered, \(S = \sum_{i=1}^{m}(e_{i} - \bar{e})^2\), \(e_{i} = \sum_{k=1}^{r}e_{ik}\) (the last column of Table 4), \(\bar{e} = \frac{\sum_{i=1}^{m}e_{i}}{m}\).

In fact, the concordance degree of experts’ estimates is determined by the value \(\chi^2\) rather than the concordance coefficient \(W\) (Kendall 1970):

\[
\chi^2 = Wr(m - 1) = \frac{12S}{rm(m + 1)}.
\]  

(3)

It has been shown (Kendall 1970) that if the value of \(\chi^2\) calculated by formula (3) is larger than its critical value \(\chi^2_{\nu, \alpha}\) taken from the distribution table of \(\chi^2\) with \(\nu = m - 1\) degree of freedom and the significance level \(\alpha\) chosen to be close to zero, then the statistical hypothesis about expert estimates’ consistency is adopted.

The concordance coefficient \(W = 0.561\) was calculated based on the data in Table 4. The value of \(\chi^2 = 62.79\) calculated by formula (3) exceeds the critical value \(\chi^2_{14, 0.05} = 14.062\) with the significance level \(\alpha = 0.05\) and \(\nu = 8 - 1 = 7\) degree of freedom (Fisher, Yates 1963). It shows that experts’ judgements are consistent and the criteria weights, calculated based on expert estimates can be used in multicriteria evaluation.

In practice, the criteria weights are usually determined by experts. A great number of weight determination methods are available. They range from the rating of criteria and direct evaluation to criteria pairwise comparison AHP (Analytic Hierarchy Process) developed by Saaty (Saaty 1980; Ginevičius et al. 2004, 2007; Brauers et al. 2007). In the present investigation, a direct method of weight determination was used, when each expert assesses the weight of a particular criterion, expressing it in per cent, so that the sum of criteria weights is equal to 40 (because the first criterion is assigned 60% of all criteria significance).

The estimates of 9 criteria provided by 16 experts are in Table 5. Based on these data, average values of each criterion’s estimates as well as the criteria weights \(\omega_i\) were calculated (as one-hundredth of the average value). The sum of the criteria weights \(\omega_i\) is equal to 0.4 (the last but one column in Table 5).

As mentioned above, the weight of the first criterion is fixed 0.6: \(\omega_1 = 0.6\).

Usually, several multicriteria evaluation methods are used simultaneously because each of them has some advantages, peculiar features and logic, objectively describing the specific character of the object investigated.

### Table 4. Criteria ranking

<table>
<thead>
<tr>
<th>Criterion No</th>
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<td>7</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>6</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td></td>
<td>92</td>
</tr>
</tbody>
</table>
The ranks obtained by different methods differ to some extent; therefore the integration of calculation results into a single complex evaluation is of theoretical and practical value.

The integration of methods and the suggestion of a compromise alternative will be correct if there is a correlation between the criteria values of particular methods. The closer the absolute value of the correlation coefficient is to unity, the more reasons are there for integrating all the multicriteria evaluation methods into a single 'pack'. It should be taken into consideration that in some cases the maximum criterion value, characterizing the leader, is the best, while in other cases the minimum criterion value is the best.

Quantitative evaluation methods are based on the matrix of the criteria, describing the compared object, statistical data or experts' estimates $R = \|R\|$ and the criteria weights $\omega_i$, $i = 1,...,m; j = 1,...,n$ (Tables 3, 4), where $m$ is the number of the criteria, $n$ – the number of the objects (alternatives) compared. When using quantitative multicriteria evaluation methods, the maximizing or minimizing character of the criteria is determined. For maximizing criteria the maximum values are the best, while for minimizing criteria the best values are the minimum ones. The criteria of multicriteria evaluation methods usually embrace non-dimensional (normalized) criteria values $\tilde{r}_{ij}$ and the respective criteria weights $\omega_i$ (Ginevicius 2008). Most methods use a special kind of initial data (criteria values) normalization or data transformation.

Methods differ in their complexity. The most widely used method is SAW (Simple Additive Weighing) (Hwang, Yoon 1981; Ginevičius, Podvezko 2006a). The criterion of the method $S_j$ expresses the idea of various quantitative multicriteria evaluation methods – the integration of the criteria values and their weights into one quantity.

The sum $S_j$ of normalized weighted values of all criteria is calculated for every $j$-th object by the formula (Hwang, Yoon 1981):

$$S_j = \sum_{i=1}^{m} \omega_i \tilde{r}_{ij},$$

where $\omega_i$ – the $i$-th criterion weight; $\tilde{r}_{ij}$ – the normalized value of this criterion for the $j$-th object ($\sum_{i=1}^{m} \omega_i = 1$).

In this case, the normalization of the initial data can be performed by the formula (Ginevičius, Podvezko 2004, 2006a):

$$\tilde{r}_{ij} = \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}},$$

where $r_{ij}$ – the $i$-th criterion value for the $j$-th object.

The best value of the criterion $S_j$ is its largest value.

The simplest of the applied methods is the sum of ranks of all the criteria (VS). The method’s criterion $V_j$ for every $j$-th object is determined by the formula (Ginevičius et al. 2006):

$$V_j = \sum_{i=1}^{m} m_{ij},$$

where $m_{ij}$ – the $i$-th criterion rank for the $j$-th object ($1 \leq m_{ij} \leq m$). The best value of the criterion $V_j$ is its smallest value. The criterion $V_j$ values depend neither on the normalization method’s initial data and their scale transformation, nor on the criteria weights $\omega_i$ ($i = 1,...,m$). However, the application of this method requires prior determination of the type of the criteria used which may be maximizing or minimizing. There is also a possibility to convert minimizing criteria to maximizing ones by the formula (Ginevičius, Podvezko 2007a):

$$\tilde{r}_{ij} = \frac{\min m_{ij}}{r_{ij}},$$

where $r_{ij}$ – the $i$-th criterion value for the $j$-th object. Then, the smallest criterion value will become the largest value equal to one.
The calculations have shown that this criterion may be used only for preliminary evaluation. However, in many cases, the results yielded by the method $I'S$, i.e. by ranking objects, do not differ considerably from those obtained by complex mathematical methods.

Another simple method is the geometric mean $\Pi_j$ of the normalized values of all the criteria (method $G1'$). It is calculated from the formula (Ginevičius, Podvezko 2007a, 2008b)

$$\Pi_j = \sqrt[n]{\prod_{i=1}^{m} \tilde{r}_{ij}}.$$  \hspace{1cm} (8)

The priority order based on formula (8) does not depend on the value of the criteria weights $\omega_j$; therefore, it is not necessary to include it into the above formula. The best value of the criterion $\Pi_j$ is its highest value.

To assess the performance of 5 considered enterprises, more advanced and complicated methods TOPSIS and VIKOR (Opricovic, Tzeng 2004; Hwang, Yoon 1981; Ginevičius, Podvezko 2004, 2006a, 2007a) were used alongside the above simple approaches. The former method can be applied to both maximizing criteria (whose minimum values are also the best).

The main integrated criterion $C_j^*$ of the method TOPSIS is calculated by the formula:

$$C_j^* = \frac{D_j^-}{D_j^* + D_j^-} \quad (j = 1, \ldots, n) \quad (0 \leq C_j^* \leq 1).$$  \hspace{1cm} (14)

The best alternative is associated with the highest value of the criterion $C_j^*$. The compared alternatives should be ranked in the descending order.

A compromise approach VIKOR (Opricovic, Tzeng 2004) also allows the stability intervals of the criteria weights to be established. Like TOPSIS, this method assesses the distance to the ideal solution but it is not so sensitive to instability of the initial data, offering compromise options in the case of conflicting criteria.

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The method uses 3 evaluation criteria: $S_j, R_j, Q_j,$

$$S_j = \sum_{i=1}^{m} \omega_i \tilde{r}_{ij}, \quad R_j = \max_i (\omega_i \tilde{r}_{ij}).$$  \hspace{1cm} (16)

The criteria $S_j$ and $R_j$ are calculated by the formulas:

$$S_j = \sum_{i=1}^{m} \omega_i \tilde{r}_{ij}, \quad R_j = \max_i (\omega_i \tilde{r}_{ij}).$$  \hspace{1cm} (17)

The main integrated criterion $Q_j$ is calculated by the formula:

$$Q_j = \nu (S_j - S^*)/ (S^* - S^*) + (1 - \nu) (R_j - R^*)/ (R^* - R^*),$$

where $S^* = \min_j S_j, S^* = \max_j S_j, R^* = \min_j R_j, R^* = \max_j R_j,$

$\nu$ make the majority criterion or the strategic weight (in this case, $\nu = 0.5$).

The best alternatives (enterprises) have the lowest values of the criteria $S_j, R_j$ and $Q_j$, implying that the considered alternatives should be ranked in an ascending order.

The value of the criterion of complex proportional evaluation method (COPRAS) (Zavadskas et al. 1994; Zavadskas, Kaklauskas 2007; Kaklauskas et al. 2006, 2007; Zavadskas et al. 2008) is defined by the formula:

$$Z_j = S_{j\max} + \frac{S_{j\min}}{S_{j\max}} \sum_{j=1}^{n} \frac{S_{j\min}}{S_{j\max}} \sum_{j=1}^{n} S_{j\min}.$$  \hspace{1cm} (19)
where \( S_{+j} = \sum_{i=1}^{m} \omega_i \bar{r}_{ij} \) is the sum of the weighted values \( \bar{r}_{ij} \) of \( j \)-th maximizing criteria (whose maximum values are the best) for all \( m \) objects. \( S_{-j} = \sum_{i=1}^{m} \omega_i \bar{r}_{-ij} \) is the same for \( j \)-th minimizing criteria (their minimum value \( S_{\text{min}} = \min_j S_{-j} \)).

COPRAS is based (5) on the initial data normalization method.

The results of the suggested multicriteria evaluation by 6 methods for external wall insulation are in Table 7. In the last column of this table, the values of the correlation coefficient, showing the correlation between the criteria values obtained by SAW and the criteria values obtained by other methods, are presented (Ginevičius, Podvezko 2008a).

The calculations have shown that there is a strong correlation between the criterion value obtained by SAW and the value obtained other wise. It is positive for GV, TOPSIS and COPRAS, whereas for the VS and VIKOR it is negative. The weakest correlation is between SAW and VS methods because the criterion values of the latter method do not depend on the criteria weights \( \omega_i \) and ranks which are calculated to the accuracy of one. A similar value of the concordance coefficient is also obtained for VIKOR method.

We can also see (Table 7) that wall insulation scenario of Ltd1 based on the methods SAW, TOPSIS, GV, VIKOR and COPRAS was ranked the first, while by the method VS it was ranked the second. The offer of Ltd2 was ranked the first according to VS, while being the second by SAW, TOPSIS, VIKOR and COPRAS. However, according to GV, the same scenario was ranked only the fourth. The offer of Ltd3 was the third based on all methods used, except for the assessment by GV, when ranked the second. The scenarios of Ltd4 and Ltd5 were the fourth and the fifth, respectively, by various methods. The ultimate rank was obtained by integrating all the methods into a single ‘pack’ (the last row in Table 7). We can see that the offer of Ltd1 gained the first place, while Ltd2 was the second, Ltd3 – the third, Ltd5 – the fourth and Ltd4 – the fifth.

4. Conclusions

The insulation of envelopes of residential and public buildings constructed during the years of Soviet power in Lithuania is poor, and this makes the renovation of these buildings an urgent problem. The analysis shows that the highest economic effect can be obtained by insulating the external walls. In this case, the economic effect is about twice that of window replacement.

The effectiveness of the external walls’ insulation depends on a number of factors. Experts mention more than 20 criteria. Not all of them are of the same importance; therefore, 9 key criteria were chosen. They are of various dimensions and change in various directions. This means that the situation is getting better when some of their values are growing, while, when the values of some other criteria are increasing, the situation is worsening. Quantitative evaluation of these complex phenomena can be successfully performed by multicriteria evaluation methods. They can be applied when the values and weights of all the criteria are known.

Methods of multicriteria evaluation were used in selecting the most economical thermal insulation for the main building of Vilnius Gediminas Technical University. The calculations were made in the framework of the international project Framework 6 „Bringing Retrofit Innovation to the Application of Public Buildings“ (BRITA in PuBs). The calculations were performed by 6 multicriteria evaluation methods, since all of them have

<table>
<thead>
<tr>
<th>Method</th>
<th>Estimate</th>
<th>Ltd1</th>
<th>Ltd2</th>
<th>Ltd3</th>
<th>Ltd4</th>
<th>Ltd5</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAW</td>
<td></td>
<td>0.2188</td>
<td>0.2050</td>
<td>0.1977</td>
<td>0.1884</td>
<td>0.1901</td>
<td>1.0</td>
</tr>
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<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOPSIS</td>
<td></td>
<td>0.745</td>
<td>0.562</td>
<td>0.392</td>
<td>0.217</td>
<td>0.201</td>
<td>0.99</td>
</tr>
<tr>
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<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GV</td>
<td></td>
<td>0.2231</td>
<td>0.1878</td>
<td>0.1905</td>
<td>0.1758</td>
<td>0.1899</td>
<td>0.89</td>
</tr>
<tr>
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<td>1</td>
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<td>2</td>
<td>5</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VS</td>
<td></td>
<td>24</td>
<td>23.5</td>
<td>28</td>
<td>31</td>
<td>28.5</td>
<td>–0.86</td>
</tr>
<tr>
<td>Rank</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>VIKOR</td>
<td></td>
<td>0.0408</td>
<td>0.176</td>
<td>0.5224</td>
<td>0.7077</td>
<td>1</td>
<td>–0.87</td>
</tr>
<tr>
<td>Rank</td>
<td>1</td>
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<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COPRAS</td>
<td></td>
<td>0.2186</td>
<td>0.2051</td>
<td>0.1978</td>
<td>0.1891</td>
<td>0.1909</td>
<td>0.99</td>
</tr>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of ranks</td>
<td>7</td>
<td>13</td>
<td>17</td>
<td>28</td>
<td>25</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Ultimate rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>
some peculiarities. To facilitate this process, the average estimate value should be found. For the integration of methods to be correct, it is necessary to determine the correlation between the values obtained by various evaluation methods.

References


Evaluation of wall insulation by multicriteria methods

R. Ginevičius et al.

PASTATŲ SIENŲ ŠILTINIMO VARIANTŲ VERTINIMAS TAIKANT DAUGIAKRITERIUS METODUS

R. Ginevičius, V. Podvezko, S. Raslanas

S a n t r a u k a


Reikšminiai žodžiai: pastatų renovacija, sienų šiltinimas, daugiakriteris vertinimas, metodų suderinamumas.